Differential Equation of wave motion

$$\because \quad y = a \sin \frac{2\pi}{\lambda}(vt - x) \rightarrow (i)$$

$$\text{velocity} = \frac{dy}{dx} = \frac{2\pi v}{\lambda} a \cos \frac{2\pi}{\lambda}(vt - x) \rightarrow (ii)$$

But $$\frac{dy}{dx} = -\frac{2\pi}{\lambda} a \cos \frac{2\pi}{\lambda}(vt - x) \rightarrow (iii)$$

from eqn (ii) & (iii)

$$\frac{dy}{dt} = -v \frac{dy}{dx} \rightarrow (iv)$$

particle velocity = wave velocity × slope of displacement curve or strain.

$$\frac{d^2y}{dt^2} = -\left(\frac{2\pi v}{\lambda}\right)^2 a \sin \frac{2\pi}{\lambda}(vt - x)$$

$$= -\left(\frac{2\pi v}{\lambda}\right)^2 y = -\omega^2 y \rightarrow (v)$$

As $\omega = 2\pi n = \frac{2\pi}{T} = \frac{2\pi v}{\lambda}$

$$\frac{d^2y}{dx^2} = -\left(\frac{2\pi}{\lambda}\right)^2 a \sin \frac{2\pi}{\lambda}(vt - x)$$

or $$\frac{d^2y}{dx^2} = -\frac{\omega^2}{v^2} \cdot y \rightarrow (vi)$$

comparing eqn (v) & (vi) we get

$$\frac{d^2y}{dx^2} = v^2 \frac{d^2y}{dx^2} \rightarrow (vii)$$

This eqn (vii) represents the differential eqn of the wave motion & its general solution is

$$y = f(vt - x) + g(vt + x)$$

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